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Fourth Grade – Standards

1. Developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends – Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

2. Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, multiplication of fractions by whole numbers – Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry – Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

**MATHMATICAL PRACTICES**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**OPERATIONS AND ALGEBRAIC THINKING**

Use the four operations with whole numbers to solve problems.

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (Note: See Glossary, Table 2.)

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “‘Add 3’ and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

**NUMBER AND OPERATIONS – FRACTIONS**

Note: Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, & 100. Extend understanding of fraction equivalence and ordering.

4.NF.1 Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

**NUMBER AND OPERATIONS IN BASE TEN**

Note: Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Generalize place value understanding for multi-digit whole numbers.

4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}; \frac{3}{8} = \frac{1}{8} + 2\frac{1}{8}; 2\frac{1}{8} = 1 + 1\frac{1}{8} = \frac{9}{8} + \frac{1}{8}$.
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$. For example, use a visual fraction model to represent $\frac{2}{3}$ as the product $2 \times (\frac{1}{3})$, and explain equivalence of fractions in terms of equally sized parts by pointing to the model (This is a whole number times a fraction analogy that is intended for students who haven't learned to multiply fractions).
   b. Understand a multiple of $\frac{a}{b}$ as a multiple of $\frac{1}{b}$, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express $\frac{3}{8} \times 4$ as $3 \times (\frac{1}{8})$, recognizing this product as $\frac{3}{2}$. (This is a multiple of a fraction analogy that is intended for students who haven't learned to multiply fractions).
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to express the problem. For example, if each person at a party will eat $\frac{3}{8}$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5 Express a fraction with denominator 10 a/b as an equivalent fraction with denominator 100, and use this technique to add fractions with respective denominators 10 and 100. (Note: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite $0.62$ as $62/100$; describe a length as $0.62$ meters; record $0.62$ on a number line diagram.

4.NF.7 Compare two decimals by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.

MEASUREMENT AND DATA

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g, lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Represent and interpret data.

4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, and the measure of an angle is the number of one-degree angles (or of 1/360 of a circle) into which it is divided. (This circle has a center that matches the common endpoint of the rays, and the degrees of an angle describe how far counterclockwise the ray on the common endpoint has rotated from the ray extending clockwise. In this version, the degrees of an angle specify the arc length on the unit circle, in which 360 degrees is the length of the entire circle).
   b. An angle that turns through $\frac{n}{2}$ of a circle is said to have an angle measure of $n$ degrees. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the parts is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

GEOMETRY

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Draw points, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Multi-Step Multiplication

Common Core Standard:
Use the four operations with whole numbers to solve problems.
4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Additional/Supporting Standard(s):
4.NBT.4, 4.NBT.5, 4.NBT.6, 4.NBT.7 Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standards for Mathematical Practice:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

Student Outcomes:
• I can make reasonable estimates with two-step story problems
• I can solve two-step problems using a variety of strategies
• I can justify my reasoning for estimating and solving problems

Materials:
• Two-Step task cards (1 set per group of 3-4 students)
• Student journals, blank or graph paper or individual white boards and markers

Advance Preparation:
• Teacher has story problems ready to display for the class or on individual sheets for students
• Teacher has sets of task cards cut apart and bagged for groups
• Students should be familiar with estimating sums using multiple strategies
• Students should be familiar with using multiple strategies for multiplication and explaining their strategies
• During small group work students should be grouped with others with like ability in math

Directions:
1. The teacher will present the problem on the board or has it printed so each student has their own copy: The class is filling bags of popcorn for the school’s Fall Carnival. Mary has filled 3 bags of popcorn. Thomas has filled 9 times as many as Mary. Amy has filled 5 times the amount of bags as Thomas. What is the overall total amount of popcorn bags that have been filled?
2. Teacher asks students to estimate mentally the total number of popcorn bags. Ask volunteers to share different methods of mental computation for this problem, and record their methods on the board. Students explain their reasoning.

   Ex.1. 3 x 10 = 30  (Thomas has about 30 bags)
          30 x 5 = 150 (Amy has about 150 bags)
          150 + 30 + 0 = 180 (I did not add Mary’s bags since it was just 3)

   Ex.2. 3 x 9 = 27 (I know 3x9 is 27 so Thomas has 27 bags. 27 is close to 30)
          30 x 5 = 150 (Amy has about 150)
          150 + (30 + 3) = 183 (I added Mary’s 3 bags to Thomas’ total so they have about 183 bags.)

3. Once a couple ways to estimate have been discussed, students solve problem finding the exact answer on their own using a strategy that makes sense to them.

4. Once finished, partners share their strategies.

5. Teacher facilitates a whole class discussion. Have several students with different strategies share their thinking using a document camera or white board for the whole class. Have students look at how the strategies are alike or different. Make connections with their estimates. How did estimating first help with solving the problem?

6. Present a second problem for students to estimate mentally.

   Ryan, Tom and Jordan collect baseball cards and are getting ready to make some trades. Ryan has 43 cards and Tom has twice as many as Ryan. Jordan has 50 more cards than Ryan and Tom have together. How many cards are available to trade?

7. Repeat steps 2-5.

8. Teacher divides the class into groups of 3-4 to use the format of “Show Down” to solve problems:
   - Teacher gives one student (student leader) in each group a set of two-step problems cards.
   - The student shows and reads the problem to the group
   - Individually students write an estimate to the problem either on a small white board or journal page (keeping their estimate to themselves)
   - When all students are ready, the student leader says “Show Down”, all other students show their estimates
   - The student leader has each group member explain how they arrived at their estimate
   - Students can use their white board to explain their thinking to the group

9. Once all in the group have shared, the student leader has the group solve the problem by finding the exact answer. When all are finished, the student leader says “Show Down”. All other students show their strategy for solving the problem. The group discusses the correct answer and strategies.

10. A new leader repeats the process. Students create their own multi-step problems to use for Show Down.
Questions to Pose:
As students work individually or with small groups:
  • Can you describe your method to me? Can you explain why it works?
  • How did you get your answer?

During whole group discussion:
  • How does your strategy relate to…?
  • Why did you decide to use this method?
  • Do you think this strategy will work with other numbers?

Possible Misconceptions/Suggestions:

<table>
<thead>
<tr>
<th>Possible Misconceptions</th>
<th>Suggestions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students have difficulty estimating for a two-step problem.</td>
<td>Students practice estimating with one step problems.</td>
</tr>
<tr>
<td>Students have difficulty solving two-step problems.</td>
<td>If students can solve one-step problems, help them break the two-step problems into the two separate steps.</td>
</tr>
<tr>
<td></td>
<td>If students struggle with one-step problems, practice solving one-step problems guiding them to use problem solving strategies such as visualizing, making sense of the problem, drawing, etc.</td>
</tr>
</tbody>
</table>

Special Notes:
Students work in pairs to develop their own two-step problems to be used to play Show Down

Solutions:

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible Estimates</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole class problem 1</td>
<td>160-190</td>
<td>165 bags of popcorn</td>
</tr>
<tr>
<td>Whole class problem 2</td>
<td>290-320</td>
<td>308 baseball cards</td>
</tr>
<tr>
<td>Task 1</td>
<td>300-360</td>
<td>346 books</td>
</tr>
<tr>
<td>Task 2</td>
<td>4000-4300</td>
<td>4200 people</td>
</tr>
<tr>
<td>Task 3</td>
<td>50-60</td>
<td>56 blocks</td>
</tr>
<tr>
<td>Task 4</td>
<td>20-30</td>
<td>25 cans</td>
</tr>
</tbody>
</table>
## Task Cards

1. The school library has 286 books. If the school librarian buys 12 books each month for five months, how many books will the library have in all?

2. On Friday, 1,050 people visited the zoo. Three times as many people visited on Saturday than on Friday. How many people visited the zoo on Friday and Saturday?

3. Nyasia always takes the same route when she walks her dog. First, she walks 7 blocks to the park. Then she walks 9 blocks to the elementary school. Finally, she walks 12 blocks to get back home. Nyasia walks her dog 2 times each day. How many blocks does Nyasia's dog walk each day?

4. Julian bought 9 packages of cat food and 5 packages of dog food. Each package of cat food contained 5 cans, and each package of dog food contained 4 cans. How many more cans of cat food than dog food did Julian buy?
Building 10,000

Common Core Standard:
Generalize place value understanding for multi-digit whole numbers.
4.NBT.1 Recognize that in a multi-digit whole number, a digit in the one place represents ten times what it represents in the place to its right. For example, recognize that 70 ÷ 7 = 10 by applying concepts of place value and division.

Additional/Supporting Standard:
4.NBT.4 Generalize place value understanding for multi-digit whole numbers.

Standards for Mathematical Practice:
2. Reason abstractly and quantitatively
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:
• I can use models to reason about place value.
• I can build numbers using place value.

Materials:
• Base ten blocks, enough to build ten 1000 cubes. If you don’t have enough use square boxes (such as tissue boxes) or make tagboard 1000 cubes if. You may also want paper copies of 100 flats to cover faces of the boxes.

Advance Preparation:
• Gather base ten blocks, enough for groups of 4 to have:
  - ten 1s, ten longs (rods), ten flats and one 1000 cube
  - Have available materials to build 1000 cubes if needed
• Students need to be familiar with base ten materials

Directions:
1. Begin with a class discussion using the unit cubes of base ten blocks. Discuss the cube shape. Continue with a review that it takes 10 cubes to make a rod. 10 ones or 10 × 1 is 10. Discuss the shape. This is a long or a rod. Record on the board:
2. Ask the students “What do you think comes next?” When a student says 100, bring out a 100 flat. It takes ten rods to make a flat. Have students talk about the shape of the flat and 100 being $10 \times 10$. Record:

3. Review the shapes (cube, long, square), and numbers, (1, 10, 100). At this point the teacher can have the table groups set up ten 1s and place it on top of a long to prove that ten 1s is also one 10. This can be repeated with ten longs placed on top of one flat. The teacher uses the language of ten 1s time 10 is one 10. Ten 10’s times 10 is one hundred.

4. Ask “What comes next?” Students will say 1000 or ten 100’s. At this point, students build ten 100s with ten flats. Students may stack them on top to make a cube or they may lay them out next to one another.

5. Once groups have made 1000, bring the group back to look at the structure and pattern of the numbers that have been written on the board. Use a student example of ten 100s stacked to make a cube. If a group does not have a 1000 cube built, introduce the 1000 cube base-ten block. Draw attention to the shape structure: cube, long, flat, cube. With every three places, the shapes repeat. Each cube represents a 1, and each long represents a 10, and each flat represents a 100.

6. Have students discuss what comes next. Emphasize the patterns they see. Once students agree that ten $\times$ 1000 comes next and it could be a long, have students discuss how the class could build 10,000.
   - Students could use base-ten 1000 cubes along with thousand cubes made from 100 flats.
   - Students could make a 1000 cube model from a square tissue box, gluing a paper copy of a flat on each face of the cube. Then put 10 together.
   - Students could make a 1000 cube model from cutting poster board, gluing a paper copy of a flat on each face of the cube
   - Students could use a combination of the techniques listed above
7. Once students have built 10,000, they arrange the cubes side by side to form a long strip of cubes. Depending on space, this could be done in a hall or on the playground.

8. Ask, “What comes next?” When students discover it is ten \times 10,000 and the shape is a square, discuss how the class could build 100,000.
   - Students could measure the length of the 10,000 model. Using string or something similar, students mark off a square that would represent the 100,000 model. (building the 100,000 model is optional)

9. To conclude, the class will discuss connections to the size and magnitude of each model. See questions below.

Questions to Pose:
As students are working:
   - How does the long (10) relate to flat (100)?
   - What patterns or structure do you notice?
   - How would knowing the structure of number help you?
   - What is the relationship in size between the long (10,000) and the flat (100)?
   - What have you learned from building these numbers?

Possible Misconceptions/Suggestions:

<table>
<thead>
<tr>
<th>Possible Misconceptions</th>
<th>Suggestions</th>
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<tbody>
<tr>
<td>Students have difficulty understanding a digit in the one place represents ten times what it represents in the place to its right.</td>
<td>Students work with models to build numbers.</td>
</tr>
<tr>
<td></td>
<td>20 is ( \times ) 10. Build a set of 2 and repeat it 10 times. Continue with 200. Build a set of 20 and repeat it 10 times.</td>
</tr>
</tbody>
</table>

Special Notes:
   - This lesson could go over two days.
   - The teacher should take pictures of the models at different stages.
   - The teacher should make the connection between the model and where it would go on a number place value chart.
   - Students can also build 1000s by making ten 100s charts and taping together. How many 1000 strips would be needed to make 10,000? How about 100,000? What would come next? How could we build the next number?
   - Use Arrow cards to build numbers: [http://www.educationworld.com/a_curr/mathchat/mathchat024.shtml](http://www.educationworld.com/a_curr/mathchat/mathchat024.shtml)

Solutions: N/A

Adapted from *Teaching Student-Centered Mathematics Grades 3-5* Van de Walle and Lovin (2006)
Build A Number

Common Core Standard:

Generalize place value understanding for multi-digit whole numbers.
4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Additional/Supporting Standard(s):

4.NBT.1 Generalize place value understanding for multi-digit whole numbers.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:

• I can determine the value of a number by its place in the number
• I can accurately read and write multi-digit whole numbers

Materials:

• One set of large digit cards, 0-9 for class demonstration
• One set of small digit cards, 0-9 for each student
• Build A Number Worksheet

Advance Preparation:

• Make one set large digit cards, 0-9 for class demonstration.
• Copy, cut and bag sets of small digit cards, 0-9 for each student
• Copies of Build A Number Worksheet, one per student
• Students have experience working with place value
• Consider grouping students for small group and partner work

Directions:

Whole Class:
1. Select 4-6 students to hold demonstration digit cards in front of the class for all to see
2. Students seated model the same digits at their seat with a small set of digit cards.
3. Teacher says a 4-6 digit number aloud. Students with the demo cards arrange themselves to create that number. Students at their seats use their small digit cards to create the number.
4. Teacher has students say the number in words.
5. Teacher has students point to the digit in the hundreds place, tens place etc. At this time, students with the demo cards hold the card up.
6. Repeat multiple times to assess place value concepts. Teacher should change digits and students to demonstrate in the front of the classroom.
7. Teacher will change from calling numbers to number clues.
   • Build the largest number you can

...
• Build a number less than 4803
• Build a number greater than 3750 and less than 3900
• Build a number 100 more than 1834

8. After students have had sufficient practice, they should be given the Build A Number Worksheet to choose their own digits. The number of digits will depend on student ability.

9. Students write 5-8 Build a Number clues
• Students choose the digits they want to use and record them on their sheet
• Student write a clue in box 1 to build a number that uses the chosen digits
• In box 2, students write a clue that will build a new number but will still use the digits chosen on the sheet.

Digits: 3 4 0 7 5

| 1. My number is a 4 digit number greater than 7000 and less than 7500. | 2. My number is an even number between 3705 and 3800 |

• Students create an answer key on the back of their paper. Some clues will have multiple answers or a range of answers.

**Partner/small group:**
1. Using their own Build A Number Worksheets and sets of small digit cards, students work in partners or a small groups to create numbers. One student reads their clue and other student in the group builds the number. After one student reads 1-3 clues, another student takes a turn at asking clues to match their chosen digits.

**Questions to Pose:**

**During Whole Class:**
• What strategies did you use to figure out the value of the digit?
• What would your number be if you added 10 more? 100 more? 1000 more?
• What would your number be if you subtracted 10? 100? 1000?
• What are some other numbers that would fit the clues

**During small group/partner work:**
• How do you know your number fits the clue?
• What are some other numbers that fit the clue?
• If I had (choose a number that does not fit clue) would it fit the clue? Why not?
**Possible Misconceptions/Suggestions:**

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>Students have difficulty reading the number</td>
<td>Provide students with a place value chart with the number words written on it for support.</td>
</tr>
<tr>
<td>Students have difficulty placing the zero digit correctly</td>
<td>Provide students with a place value chart with the number words written on it for support.</td>
</tr>
<tr>
<td></td>
<td>Provide base-ten models to build the number.</td>
</tr>
</tbody>
</table>

**Special Notes:**

Students need to play Build A Number long enough to be successful in small groups.

**Solutions:** N/A

*Adapted from mathwire.com*
# Build A Number

**Choose Your Digits**

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Create an answer key on the back of this sheet.

*Adapted from mathwire.com*
Multiply Using the Distributive Property

Common Core Standard:
Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Additional/Supporting Standards: 4.NBT.2, 4.OA.1

Standards for Mathematical Practice:
1. Make sense of problems and persevere in solving them
4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:
- I can use multiple representations for multiplication
- I can use the distributive property in multiplication
- I can explain the distributive property and why it works

Materials:
- Base ten blocks (at least 20 rods and 30 unit cubes per pair)
- Recording Sheet (one per student)

Advance Preparation:
- Gather base ten blocks. You may want to create a bag of blocks for each pair to make them easier to distribute.
- Make copies of the Recording Sheet
- Consider how students should be paired or grouped for this lesson
- Students should be familiar with mental computation and explaining their strategies
- Students should be able to flexibly record large numbers in multiple forms including expanded notation
- Students should understand that multiplication is a representation of equal groups
Directions:

1. Begin the lesson by writing $6 \times 32$ for all the students to see. Ask the students to compute $6 \times 32$ mentally without paper, pencil, or calculators. Ask volunteers to share different methods of mental computation for this problem, and record their methods on the board. Spotlight any method that uses the distributive property:

Example of a response that uses the distributive property:

$6 \times 32 = 6 \times 30 + 6 \times 2$

$6 \times 30 = 180$ and $6 \times 2 = 12$

$180 + 12 = 192$

Point out how the number 32 was broken into 30 + 2. This makes it easier to multiply and add mentally. Thus, this problem could be written as follows:

$6 \times 32 = (6 \times 30) + (6 \times 2)$

2. Using base ten materials (such as blocks), students should model $6 \times 32$. This task should be completed independently or with a neighbor by showing 6 groups of 32 as shown below:

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3. Using the base ten blocks, guide students to observe that they have 6 groups of 30 and 6 groups of 2. Ask the students, “How would these groups look in a number expression?” Help students make a connection between the blocks and the expression $(6 \times 30) + (6 \times 2)$

4. Have students record their drawings and solution in symbols on their Recording Sheets.

5. Present students with another problem $4 \times 53$ by recording it on the board. Ask the students to model this problem with base ten blocks. Have them draw their diagram on the Recording Sheet. Then work with a partner to show this problem using numbers and symbols. Students should take turns explaining the mathematics they are using and why it makes sense. Once all have explained the problem to their partners, write the steps on the board.

6. When most pairs have finished, ask students to share their thinking about the mathematics they are using. Based on student explanations, introduce the term “Distributive Property.” Ask student to explain how the distributive property is related to expanded notation. As students share their thinking record any method that makes this connection such as:

$53 = 50 + 3$  So  $4 \times 53 = (4 \times 50) + (4 \times 3)$

7. Once again, present a third problem such as $3 \times 67$ by writing it on the board. Ask students to solve the problem by grouping their blocks. They should use the same process of modeling the problem with blocks and recording their work on the Recording Sheet. This time, challenge students to use the term “distributive property” as they explain the mathematics they used with their partners.
8. When most pairs have finished, ask students to share the mathematics they used with the whole class. Challenge them to use the term “distributive property” as they explain their reasoning.

9. Finally students will apply their understanding in the context of a word problem such as: A candy company has orders for chocolate bars from 5 different stores. Each order contains 45 chocolate bars. Use the distributive property to figure out how many chocolate bars the candy company needs to make.

Students should solve this problem using the same process: modeling with blocks, recording on the Recording Sheet, and discussing their reasoning with a partner.

10. Once most pairs have completed the task, close the lesson by asking students to explain how using the distributive property can help you solve multiplication problems. This question might prompt a small group or whole-class discussion or it might be used as a journal prompt.

**Questions to Pose:**
As students work with their partner:
• How does your model/drawing connect to the numbers and symbols?
• Why does it make sense to use an addition symbol in the expression?
• How does breaking the number apart help you solve the problem?
• How did you decide where to start solving this problem?

During class discussion:
• Explain how the distributive property is helpful when you are multiplying large numbers. Create a multiplication problem that supports your explanation.
• What would happen if you tried to use the distributive property to solve a problem such as 458 x 7?
• How does the distributive property connect to the models we have been using?

**Possible Misconceptions/Suggestions:**

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<tr>
<th>Possible Misconceptions</th>
<th>Suggestions</th>
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<tbody>
<tr>
<td>Students have difficulty decomposing numbers using place value. They have trouble recognizing that 32 is the same as 30 + 2.</td>
<td>Give students the opportunity to build two-digit numbers with a variety of manipulatives. Emphasize the values of tens and ones.</td>
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<tr>
<td>Students do not connect multiplication with equal groups.</td>
<td>Provide a variety of manipulatives for students to partition into equal groups. Encourage students to provide stories that match their manipulatives such as “Four squirrels each carried 8 acorns.” Help students write a multiplication expression that corresponds with their manipulatives and context.</td>
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</table>
Special Notes:
Follow up lessons could

- Extend to multiplying with larger numbers including three digits by one digit and four digits by one digit
- Vary the problem type based on Table 2 “Common Multiplication and Division Situations.” This table is found on page 89 of the Common Core State Standards for Mathematics Glossary. [http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf](http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf)
- This lesson applies the distributive property to decomposing large numbers (32 = 30+2). Provide problems that model and apply the distributive property in reverse such as 4 x (8 + 5)
- Explore whether or not the distributive property could be used in addition, subtraction, and division

Solutions: NA

Adapted from “Exploring the Distributive Property” Mathematics Enhanced Scope and Sequence, Virginia Department of Education, 2011.
Multiply Using the Distributive Property
Recording Sheet

Name ________________________________ Date ____________________

<table>
<thead>
<tr>
<th>Pictures</th>
<th>Numbers and Symbols</th>
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| (Use ☐ for tens and □ for ones.) | 6\times32=(\_\times\_)+ (\_\times\_)

Strategies for Multiplying Multi-digit Numbers

Common Core Standard:
Use place value understanding and properties of operations to perform multi-digit arithmetic.
4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Additional/Supporting Standards:
4.OA.1, 4.OA.2 Use the four operations with whole numbers to solve problems.

Standards for Mathematical Practice:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:
• I can develop and implement a strategy for solving a multi-digit multiplication problem
• I can use a variety of strategies to solve multi-digit multiplication problems
• I can explain my reasoning when developing a multiplication strategy

Materials:
• Outdoor Adventure Store Recording Sheet (one per student)

Advance Preparation:
• Copy the Outdoor Adventure Store Recording Sheet
• Students should be comfortable with a variety of strategies for one-digit multiplication
• Students should have a strong understanding of the meaning of multiplication, especially the use of equal groups.

Directions:
1. Distribute the Outdoor Adventure Store Recording Sheet and ask students to read and solve the first problem.

2. Allow students to struggle. The students should be devising their own approaches to solve the problem. Some will draw pictures, others will add, some will use concepts of multiplication.

3. As you circulate note which strategies the students are using. Take note of common strategies, unique strategies, simple strategies, and strategies that are problematic.

4. Determine which students you will ask to share. Remember that your goal is to help students develop effective place-value based strategies that illustrate the meaning of multiplication. This is not a time to study the standard algorithm.
5. When most students have finished the first problem, invite several to share their strategies. Try to examine every type of strategy used in the room. At this point **do not** share the standard algorithm.

Sharing Procedure
1. Display the student’s strategy. You may use a document camera or have the student copy their work onto the board.
2. Ask the student to explain their reasoning for each step.
3. Ask questions to determine the student’s level of understanding, “Why did you…”
4. Allow the class to ask the student questions about the strategy.
5. To ensure others are listening, have each student explain the strategy to a neighbor.
6. Make any clarifying statements about the strategy.
7. Invite a new student to share a different strategy.

6. Once several strategies have been shared using the above process, direct the students to complete problem #2. This task requires the students to solve the same problem using a new approach. Encourage the students to use one of the approaches shared by a classmate.

7. Again, ask a few students to share. This sharing session can be much shorter because it is a second-look at the same multiplication problem.

8. Now move on the question #3 on the Recording Sheet and repeat the process described in steps 2-7.

9. To close the lesson, ask students to think about their favorite strategy from today. The students could discuss this strategy and why they like it with a partner or they could write their response in a journal.

**Questions to Pose:**
- Why did you…?
- How is Latisha’s strategy similar to Jonah’s? How are they different?
- Where did you run into difficulty in your work? Why was it problematic?
- Why is it important that your strategies are accurate and efficient?
- Which strategy is most efficient? Which strategy is least efficient?
### Possible Misconceptions/Suggestions:

<table>
<thead>
<tr>
<th>Possible Misconceptions</th>
<th>Suggestions</th>
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</thead>
<tbody>
<tr>
<td>Students have difficulty developing their own strategy</td>
<td>Ask students to think about the strategies they could use for a simple problem such as $3 \times 4$. Connect these strategies to the problems in today’s lesson.</td>
</tr>
<tr>
<td>Students insist on using the standard algorithm which they learned outside of school</td>
<td>Explain that they may be able to use the standard algorithm in the future, but they must first be able to explain why the steps make sense. The purpose of today’s lesson is to explore the meaning of multiplication in multi-digit numbers. Later they may use these strategies to help them understand the standard algorithm, but not today. The standard algorithm for multiplication is found in 5th grade.</td>
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### Special Notes:
- This is an introductory lesson to multiplying with multi-digit numbers. Students should develop their own strategies. If a student uses the standard algorithm because it was learned at home, suggest that she attempt a different strategy. Note the suggestion listed in the box above.

### Solutions:
- $25 \times 14 = 350$
- $21 \times 7 = 147$
Outdoor Adventure Store Recording Sheet

Directions: Use what you know about multiplication to solve the following problems from the Outdoor Adventure Store.

1. Luis sells 25 hiking packs per day at the Outdoor Adventure Store. How many packs will he sell in 14 days?

2. Solve problem #1 using a different strategy.
3. Central Elementary School orders 21 packs of tablecloths for a school picnic. Each pack contains 7 tablecloths. How many tablecloths will they have for the picnic?

4. Solve problem #3 using a different strategy
Fraction Card Games

Common Core Standard:

Extend understanding of fraction equivalence and ordering.

4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Additional/Supporting Standards:

4.NF.1 Extend understanding of fraction equivalence and ordering

Standards for Mathematical Practice:

2. Reason abstractly and quantitatively
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:

• I can compare two fractions with different numerators and/or denominators
• I can recognize equivalent fractions

Materials:

• Fraction Cards sets (1 set/pair) 38 cards in the set

Advance Preparation:

• Copy Fraction Cards on cardstock
• Consider how you will group students
• Students need to be familiar with fraction benchmarks such as ½
• Students need to understand fractions as parts of whole
• Students need to be familiar with finding equivalent fractions
• Students need to understand the larger the denominator, the smaller the parts

Directions:

Game 1: Concentration (2-3 students)

• Deal cards face down in five rows
• Players take turns turning over two cards at a time
• If the fractions are equivalent, the student keeps the pair
• The winner is the person with the most cards

Game 2: Go Fish (2-3 students)

• Deal five cards to each player, stack the rest of the cards in the middle of the table
• The object is to get pairs of equivalent fractions
• At each turn players may ask others in the group for a certain fraction
• As long as someone gives the person a card, the player may keep asking
• When no one has an equivalent fraction to give the player, the person ‘goes fishing’ by drawing from the deck
• At end of game, the player with the most pairs wins

Game 3: War (2 students)
• Cards are divided between two players
• Each player lays down 1 card
• Players decide which fraction is greatest
  - Players may create common denominators
  - Compare to a benchmark fraction such as $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$
• Player with the largest fraction picks up both cards
• If fractions are equivalent, players lay down a second card and compare
• At the end of the game the player with the most cards wins

Questions to Pose:
As students are playing games:
• What strategies are you going to use to figure out the value of the fraction?
• How might drawing a picture be helpful
• Give me an example of a fraction that is less than…greater than…equivalent to…?
• Describe the method you used to compare the fraction? Explain why it works?
• Is your fraction close to a benchmark? How can you tell?
• What are the benefits of using a common denominator to compare fractions?

Possible Misconceptions/Suggestions:

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<tbody>
<tr>
<td>Students have difficulty seeing equivalent fractions</td>
<td>Students work with models such as pattern blocks, fraction strips, fraction circles and number lines to explore equivalent fractions, 1 blue parallelogram is 1/3 and 2 triangles are 1/3 of a hexagon</td>
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<td>Students do not use benchmark fractions when comparing fractions</td>
<td>As teacher circulates to monitor student understanding, ask: Is your fraction close to a benchmark number? How can you tell? Give student different lengths of paper strips. Student folds in half then fourths. Student labels strips with $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$</td>
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<tr>
<td>Students have difficulty comparing fractions with different denominators</td>
<td>Work with student to find common denominators when comparing fractions</td>
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Special Notes:
Games need to be played multiple times. Classroom discussions after students play games should focus on strategies for efficiently comparing fractions using common denominators and benchmarks.

Solutions: N/A
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NC DEPARTMENT OF PUBLIC INSTRUCTION
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Diagram:
- Octagon divided into 8 equal parts, with 1 part shaded.
- Grid with 9 squares, with 3 squares shaded.
Give ‘Em Chocolate!

Common Core Standard
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
4.NF.3 Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1/8 = 8/8 + 8/8 + 1/8.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Additional/Supporting Standard:
4.NF.3.c Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Standards for Mathematical Practice:
1. Make sense of problems and persevere in solving them
4. Model with mathematics
5. Use appropriate tools strategically
8. Look for and express regularity in repeated reasoning

Student Outcomes:
• I can decompose a fraction into a sum of fractions with the same denominator
• I can record each decomposition using an equation
• I can use models to demonstrate why my equation makes sense

Materials:
• Student manipulatives for twelfths such as fraction bars

Advance Preparation:
• Gather fraction manipulatives
• Copy the Record Sheet (optional)
• Consider how you will partner students
• Students should have an understanding of unit fractions as they relate to the whole.
Directions:
1. In this task students apply their understanding of unit fractions to a real-world context.

2. Describe the following situation for the students:
   
   You bought 12 pounds of chocolate to give to 5 people as gifts. You want to give away all 12 pounds to the 5 people. What are the different ways I can partition my chocolate?

3. Solicit student suggestions. For instance: \[\frac{2}{12} + \frac{4}{12} + \frac{3}{12} + \frac{1}{12} + \frac{2}{12}\]
   Students may note that these are not equal portions.

4. Distribute the Recording Sheet or have the students create their own. Ensure that students understand the expectations of the task:
   - Work in pairs to come up with as many ways to partition the 12 pounds of chocolate as possible.
   - Record each method using an equation
   - Prove each method using a visual model such as a sketch or with manipulatives

5. Circulate as pairs of students work together on the task. As you circulate ask questions including those listed below.

6. To close the lesson, invite each group to present one way to partition the chocolate. Their presentation should include both an equation and visual model

Questions to Pose:
Before:
- What visual models could you use to represent your solutions?
- How might manipulatives help you solve this problem?

During:
- Describe the method you used to partition the chocolate?
- How does your visual model represent this solution?
- How does your equation represent this solution?
- How do the unit fractions relate to the whole?

After:
- What did you notice about partitioning the whole?
- What surprised you as you completed this task?
- How did working with unit fractions support your reasoning?
- How did the visual models and equations support your thinking?
- Are there any solutions that we’ve missed?
**Possible Misconceptions/Suggestions:**

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<thead>
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<th>Possible Misconceptions</th>
<th>Suggestions</th>
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</thead>
<tbody>
<tr>
<td>Students have difficulty breaking 12/12 into five parts.</td>
<td>Use a twelfths model such as bars, strips, or circles. Work with the student to physically separate the twelve parts into 5 groups. Guide the student as they record the fractional parts in an equation.</td>
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<tr>
<td>Students have trouble connecting the visual models, equations, and story problem.</td>
<td>Suggest that the student label the components with each of the 5 people. For instance: &lt;br&gt;<img src="https://via.placeholder.com/150" alt="Diagram" /> 3/12  &lt;br&gt;Mary Mary</td>
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**Special Notes:**

This task should be conducted after students have an understanding of unit fractions as they relate to the whole.

**Solutions:**

There are many solutions. Some solutions include:

- $\frac{3}{12} + \frac{3}{12} + \frac{3}{12} + \frac{2}{12} + \frac{1}{12}$
- $\frac{8}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$
- $\frac{7}{12} + \frac{2}{12} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$
Give ‘Em Chocolate

You bought 12 pounds of chocolate to give to 5 people as gifts. You want to give away all 12 pounds to the 5 people. What are the different ways I can partition my chocolate?

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<th>Visual Model</th>
<th>Equation</th>
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Unit Fractions

Common Core Standard:
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3 Understand a fraction $\frac{a}{b}$ with $a > 1$ as a sum of fractions $\frac{1}{b}$.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$; $\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$; $2 \frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$.

Additional/Supporting Standard:
4.NF.3.c, 4.NF.3.d Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

Standards for Mathematical Practice:
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:
• I can model a unit fraction from a whole
• I can add and subtract unit fractions from a whole
• I can decompose a whole into its units

Materials:
• Construction paper, 2 colors cut into 18” by 2” strips. (3 strips per student)

Advance Preparation:
• Cut strips of construction paper into equal size strips. Enough for each student to have 2 strips of one color and 1 strip of a second color
• Students need to be familiar with fractions being parts of a whole
• Students need to be familiar with working with a number line

Directions:
1. Teacher can have an activator or literature to tap prior knowledge of fractions
2. Students are given 2 strips of construction paper. At this point use only one color.
3. Students fold both strips into 8ths by folding it in half, in half again, in half a third time.
4. After each fold, ask student to tell their neighbor how many sections the strip will have.
5. After the 3 folds, students open the strip to find 8 sections. Have students talk about what they notice with the strip. (8 parts, there are 4 parts in each half, each section is $\frac{1}{8}$, etc.)
6. Students repeat this process with the second strip of the same color.
7. Students cut apart all 16 sections and label each section with 1/8 and place in a pile on their table.
8. Teacher gives each student a third strip that is a different color, but the same size as the two previous strips. This strip does not need to be folded. This strip will be the ‘whole’.
9. Students place the whole on the table and get their eighths ready.
10. Teacher will have student work with the eighths placing them on top or directly under the whole. Pose questions such as:
   - Build 3 eighths, build 7 eighths, build 10 eighths, build 1 and 5 eighths
   - Add 2 eighths and 5 eighths, add 5 eighths and 5 eighths
   - Subtract 3 eighths from 9 eighths
   - Using partners, one partner builds 7 eighths and one partner builds 12 eighths, what do you notice.
   - Have students build an amount of eighths such as 6 eighths, ask what are the different ways 6/8 can be broken apart (2/8 + 4/8, or 3/8 + 3/8)
11. As students use the unit fractions to build, add, subtract, decompose and compare numbers, pose additional questions. See below.
12. Students summarize learning by a journal entry: How does the unit fraction relate to whole? Use both words and pictures in your entry.

Questions to Pose:
As students are working:
- How does this unit fraction relate to the whole?
- What fraction is 2 units less than what you have? Show me with your pieces.
- Give me an example of a fraction that is less than…greater than…what you have.
- Describe the method you used to compare the fraction.
- Is your fraction close to a benchmark? How do you know?
- Describe the method you used to add /the unit fractions.
- What would your fraction look like using the symbols?
  (3 1/8 units could be 1/8 + 1/8 + 1/8 or 3/8)

Possible Misconceptions/Suggestions:
<table>
<thead>
<tr>
<th>Possible Misconceptions</th>
<th>Suggestions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students have difficulty seeing the unit fraction in</td>
<td></td>
</tr>
<tr>
<td>relation to the whole.</td>
<td>Give student experiences with</td>
</tr>
<tr>
<td></td>
<td>halves and thirds</td>
</tr>
</tbody>
</table>

Special Notes:
Once students have practice working with unit fractions in eighths, have them practice with units in tenths and twelfths. Premade circles and rectangles divided into tenths and twelfths might be easier than student measuring or folding paper. Students need to see unit fractions using multiple models.

Solutions: N/A
Show What You Know: Multiple Representations of Decimals and Fractions

Common Core Standard:
Understand decimal notation for fractions, and compare decimal fractions.
4.NF.6 Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

Additional/Supporting Standards:
4.NF.5, 4.NF.7

Standards for Mathematical Practice:
2. Reason abstractly and quantitatively
4. Model with mathematics
5. Use appropriate tools strategically
7. Look for and make use of structure

Student Outcomes:
• I can create multiple representations of a decimal
• I can compare and order decimals to the hundredths
• I can use decimal notation for fractions

Materials:
• Show What You Know Sheet or blank paper (at least 2 per student)
• Decimal Grids (several sheets per class)

Advance Preparation:
• Copy Show What You Know Sheets or gather blank paper
• Copy Decimal Grids
• Consider how you will create groups of 3-4 students
• Students should be familiar with representing decimals as fractions with denominators 10 or 100.
• Students should be comfortable with multiple representations of decimals including: number lines, grids, and using base ten blocks (where □ = 1)

Directions:
1. Begin by modeling today’s task with the whole group:
   • Choose a decimal in the hundredths place. Students will determine their own decimal.
   • Complete the graphic organizer on the Show What You Know Sheet. Alternatively, students may draw their own version on blank paper. (See example below)
2. Allow students ample time to complete their own graphic organizer. Be prepared to distribute decimal grids to students who need them.
3. Hopefully most students will have time to complete the process representing at least 2 different decimals.
4. Organize the students into groups of 3-4. In their groups the students should line up their decimals from least to greatest. As you circulate, emphasize that students should be using the representations to justify their reasoning.
5. When most groups have finished ordering the decimals, choose 6-8 graphic organizers to display.
6. Through whole-class discussion students should decide how to order the decimals from least to greatest. During this discussion ask questions (see below) to ensure that students justify their thinking.
7. To summarize the lesson, ask students to choose 3 of the decimals displayed to examine using symbols: >, <, or =. Students could also include visual models (such as a number line, grid, or fraction) to support their reasoning.

Questions to Pose:
Before:
• How could you use base ten blocks to model this decimal?
• How could you use a number line to show this decimal?
During:
• How do you know that this fraction is equivalent to your decimal?
• What would happen if you changed your number line so that it showed 2 wholes instead of 1 whole?
• How can you prove that this decimal is the largest? Or smallest? Or in between these?
After:
• What other ways could you represent this decimal?
• How can you prove this fraction is equivalent to this decimal?
• How do you know this decimal is the largest? Or smallest? Or in between these?

Possible Misconceptions/Suggestions:

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<tbody>
<tr>
<td>Students have difficulty creating fractions that are equivalent to their decimal</td>
<td>Provide multiple opportunities for students to connect decimals on a grid with decimal names and fractional representations. Note that both decimals and fractions are parts of a whole.</td>
</tr>
<tr>
<td>Students struggle to order decimals</td>
<td>Refer back to the visual models created by students. Ensure that students are using the same size whole and type of representation when comparing. By comparing shaded grids, base ten blocks, or number lines students can reason why one decimal is larger or smaller than another.</td>
</tr>
</tbody>
</table>

Special Notes:
• This lesson should be used only after students have experience modeling decimals in multiple ways, including with fractions that have denominators 10 or 100.
Sample Solution:
Show What You Know

Directions: Choose a decimal to write in the center circle. Your decimal should be in the hundredths place. In each of the 4 boxes represent your decimal in another way:

• As a fraction
• In expanded notation
• Using a number line
• Shade a grid or draw base ten blocks (Your choice! Use a grid from your teacher or draw base ten blocks)
Decimal Grids

[Grids shown with black lines on a white background, each grid containing smaller squares within larger squares, representing decimal places.]
Running the Race

Common Core Standard:
Extend understanding of fraction equivalence and ordering.
4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, <, and justify the conclusions, e.g., by using a visual model.

Additional/Supporting Standard: N/A

Standards for Mathematical Practice:
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

Student Outcomes:
• I can compare decimals to the hundredths

Materials:
• Running Times for a Mile Sheet

Advance Preparation:
• Make copies of the Running Times for a Mile Sheet. 1 for each group of students
• Students are familiar with decimal notation to the hundredths
• Consider grouping students for discussion

Directions:
1. Activate student knowledge of the sport of running. Make a reference to marathons, track and field and the Olympics. Time for running is recorded in decimals notation. Have a discussion about which is the best running time: 9.35 or 9.67 minutes.
2. The teacher could time students as a class doing jumping jacks or another activity. Teacher records the time on the board. The teacher has the students repeat the activity and records the time. Which time was best? Why?
3. Present the task to the students. “Who is the best runner?” and “How can you tell?” Groups of 3-4 students are given the Running Times for a Mile Sheet. As a group, students will justify their runner is the best. Assign a runner to each group. Some groups will have the same runner, but will work separately.
4. Each group will be responsible for writing their justification. The group will present to the class. Give students opportunities for a lively debate. The goal should be consensus, everyone agrees on one runner as the best.
5. Possible justifications might include:
   Hailey – Even though she came in last on the 3rd run, she had the fastest time for the first 2 runs.
   Jackson – His running times decrease each time. He keeps improving. If they continue, he will have a lower time. He must be practicing.
Paul - He is the best because he is consistent. He was close to first every time. He was always in the 7-minute range. He was only .02 away from 1st place on the final run.

**Questions to Pose:**
As groups are working:
- What strategy are you going to use to compare these two decimals?
- How do you know…?
- How can you convince the others that your answer makes sense?
- About how far apart are these two times?

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<tr>
<td>Students have difficulty comparing decimals.</td>
<td>Support the student as they build a model of the two decimals.</td>
</tr>
</tbody>
</table>

**Special Notes:**
The main point of this lesson is to construct a viable argument. They need to justify their conclusions and communicate them to others. They need to respond to the arguments of others.

**Solutions:** N/A
## Running Times for a Mile

<table>
<thead>
<tr>
<th>Student</th>
<th>Paul</th>
<th>Jackson</th>
<th>Hailey</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time in Min. 1&lt;sup&gt;st&lt;/sup&gt; Run</td>
<td>7.5</td>
<td>8.58</td>
<td>7.44</td>
</tr>
<tr>
<td>Time in Min. 2&lt;sup&gt;nd&lt;/sup&gt; Run</td>
<td>7.84</td>
<td>8.11</td>
<td>7.83</td>
</tr>
<tr>
<td>Time in Min. 3&lt;sup&gt;rd&lt;/sup&gt; Run</td>
<td>7.6</td>
<td>7.58</td>
<td>8.68</td>
</tr>
</tbody>
</table>